Instantons and Monopoles in the Nonperturbative QCD

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Based on the dual-superconductor picture, we study the confinement physics in QCD in terms of the topological objects as monopoles in the maximally abelian (MA) gauge using the SU(2) lattice QCD. In the MA gauge, the off-diagonal gluon component is forced to be small, and hence microscopic abelian dominance on the link variable is observed in the lattice QCD for the whole region of β. By regarding the angle variable in the off-diagonal factor as a random variable, we derive the analytical formula of the off-diagonal gluon contribution $W_C^{\rm off}$ to the Wilson loop in relation with the microscopic variable of the diagonal gluon component in the MA gauge. We find that $W_C^{\rm off}$ obeys the perimeter law, which leads to abelian dominance on the string tension. To clarify the origin of abelian dominance for the long-range physics, we study the charged-gluon propagator in the MA gauge using the lattice QCD, and find that the effective mass $m_{ch} \simeq 0.9$GeV of the charged gluon is induced by the MA gauge fixing. In the MA gauge, there appears the macroscopic network of the monopole world-line covering the whole system, which would be identified as monopole condensation at a large scale. To prove monopole condensation in the field-theoretical manner, we apply the dual gauge formalism to the monopole part, and derive the inter-monopole potential from the dual Wilson loop in the MA gauge. In the monopole part, which carries the nonperturbative aspects of QCD, the dual gluon mass is evaluated as $m_B \simeq$0.5GeV in the infrared region, which is the evidence of the dual Higgs mechanism by monopole condensation. We study the action density around the QCD-monopole in the MA gauge. Around the monopole in the MA gauge, there remains the large fluctuation of off-diagonal gluons, and large cancellation occurs between the diagonal and off-diagonal action densities to keep the total QCD action finite. The charged-gluon rich region around the QCD-monopole would provide the effective monopole size as the critical scale of the abelian projected QCD. Instantons are expected to appear in the charged-gluon rich region around the monopole world-line in the MA gauge, which leads to the local correlation between monopoles and instantons.

§1. Dual Superconductor Picture for Confinement in QCD

Quantum Chromodynamics (QCD) shows the asymptotic freedom due to its non-abelian nature, and the perturbative calculation is workable in the ultraviolet region as $\mu \gg 1$GeV. On the other hand, in the infrared region as $\mu \lesssim 1$GeV, QCD exhibits nonperturbative phenomena like color confinement and dynamical chiral-symmetry breaking (DχSB) due to the strong-coupling nature. Up to now, there is no systematic promising method for the study of the nonperturbative QCD (NP-QCD) except for the lattice QCD calculation with the Monte Carlo method.

The lattice QCD simulation can be regarded as a numerical experiment based on the rigid field-theoretical framework, and provides a powerful technique for the analyses of the NP-QCD vacuum and hadrons. Owing to the great progress of the computer power in these days, the lattice QCD becomes useful not only for "reproductions of hadron properties" but also for "new predictions" like glueball properties and the QCD phase transition. The "physical understanding" for NP-

QCD is also one of the most important subjects in the lattice QCD, because the aim of theoretical physics is never to reproduce numbers but to find the "physical mechanism" hidden in phenomena ! In this paper, we study the confinement physics in QCD using both the lattice QCD and the theoretical formalism.

In 1974, Nambu proposed an interesting idea that quark confinement would be physically interpreted using the *dual version of the superconductivity.*[1] In the ordinary superconductor, Cooper-pair condensation leads to the Meissner effect, and the magnetic flux is excluded or squeezed like a quasi-one-dimensional tube as the Abrikosov vortex, where the magnetic flux is quantized topologically. On the other hand, from the Regge trajectory of hadrons and the lattice QCD results, the confinement force between the color-electric charge is characterized by the universal physical quantity of the string tension, and is brought by *one-dimensional squeezing* of the color-electric flux in the QCD vacuum. Hence, the QCD vacuum can be regarded as the dual version of the superconductor based on above similarities on the low-dimensionalization of the quantized flux between charges.

In the dual-superconductor picture for the QCD vacuum, the squeezing of the color-electric flux between quarks is realized by the *dual Meissner effect,* as the result of *condensation of color-magnetic monopoles,* which is the dual version of the electric charge as the Cooper pair. However, there are *two large gaps* between QCD and the dual-superconductor picture.[2]

(1) This picture is based on the *abelian gauge theory* subject to the Maxwell-type equations, where electro-magnetic duality is manifest, which QCD is a nonabelian gauge theory.

(2) The dual-superconductor scenario requires condensation of (color-)magnetic monopoles as the key concept, while QCD does not have such a monopole as the elementary degrees of freedom.

As the connection between QCD and the dual-superconductor scenario, 't Hooft proposed the concept of the *abelian gauge fixing,*[3] the *partial gauge fixing* which only remains abelian gauge degrees of freedom in QCD. By definition, the abelian gauge fixing reduces QCD into an abelian gauge theory, where the off-diagonal element of the gluon field behaves as a *charged matter field* similar to $W_\mu^\pm$ in the Standard Model and provides a color-electric current in terms of the residual abelian gauge symmetry. As a remarkable fact in the abelian gauge, *color-magnetic monopoles* appear as *topological objects* corresponding to the nontrivial homotopy group $\Pi_2(\mathrm{SU(N_c)}/\mathrm{U(1)}^{\mathrm{N_c}-1})$ $= \mathbf{Z}_\infty^{\mathrm{N_c}-1}$ in a similar manner to the GUT monopole.[3–6] In general, the monopole appears as a topological defect or a singularity in a constrained abelian gauge manifold embedded in the compact (and at most semi-simple) nonabelian gauge manifold.

Here, let us consider appearance of monopoles in terms of the gauge connection. In the general system including the singularities such as the Dirac string, the field strength is defined as

$$G_{\mu\nu} \equiv \frac{1}{ie}([\hat{D}_\mu, \hat{D}_\nu] - [\hat{\partial}_\mu, \hat{\partial}_\nu]), \tag{1·1}$$

which takes a form of the difference between the covariant derivative operator $\hat{D}_\mu \equiv \hat{\partial}_\mu + ieA_\mu(x)$ and the derivative operator $\hat{\partial}$ satisfying $[\hat{\partial}_\mu, f(x)] = \partial_\mu f(x)$. By the general gauge transformation with the gauge function Ω, the field strength $G_{\mu\nu}$ is

transformed as

$$G_{\mu\nu} \equiv \frac{1}{ie} \to G'_{\mu\nu} \equiv \Omega G_{\mu\nu}\Omega^\dagger = \frac{1}{ie}([\hat{D}'_\mu, \hat{D}'_\nu] - \Omega[\hat{\partial}_\mu, \hat{\partial}_\nu]\Omega^\dagger)$$
$$= \partial_\mu A'_\nu - \partial_\nu A'_\mu + ie[A'_\mu, A'_\nu] + \frac{i}{e}\Omega[\partial_\mu, \partial_\nu]\Omega^\dagger. \quad (1\cdot2)$$

The last term remains only for the *singular gauge transformation*, and can provide the Dirac string in the abelian gauge sector. For a singular SU(N_c) gauge function, the last term leads to breaking of the abelian Bianchi identity and monopoles in the abelian gauge.

Thus, *by the abelian gauge fixing, QCD is reduced into an abelian gauge theory including both the electric current* j_μ *and the magnetic current* k_μ*,* which is expected to provide the theoretical basis of the dual-superconductor scheme for the confinement mechanism.

§2. Maximally Abelian Gauge and Abelian Projection Rate

The abelian gauge fixing is a *partial gauge fixing* defined so as to diagonalize a suitable gauge-depending variable $\Phi[A_\mu(x)]$, and the gauge group $G \equiv \mathrm{SU}(N_c)_{\mathrm{local}}$ is reduced into $H \equiv \mathrm{U}(1)^{N_c-1}$ in the abelian gauge. Here, $\Phi[A_\mu(x)]$ can be regarded as a *composite Higgs field* to determine the gauge fixing on G/H.

The *maximally abelian (MA) gauge* is a special abelian gauge so as to minimize the off-diagonal part of the gluon field,[2]

$$R_{\mathrm{off}}[A_\mu(\cdot)] \equiv \int d^4x \mathrm{tr}[\hat{D}_\mu, \vec{H}][\hat{D}^\mu, \vec{H}]^\dagger = \frac{e^2}{2}\int d^4x \sum_\alpha |A_\mu^\alpha(x)|^2 \quad (2\cdot1)$$

with the SU(N_c) covariant derivative $\hat{D}_\mu \equiv \hat{\partial}_\mu + ieA_\mu$ and the Cartan decomposition $A_\mu(x) = \vec{A}_\mu(x)\cdot\vec{H} + \sum_\alpha A^\alpha(x)E^\alpha$. Thus, *in the MA gauge, the off-diagonal gluon component is forced to be as small as possible by the gauge transformation, and therefore the gluon field* $A_\mu(x) \equiv A_\mu^a(x)T^a$ *mostly approaches the abelian gauge field* $\vec{A}_\mu(x)\cdot\vec{H}$.

Since R_{off} is gauge-transformed by $\Omega \in G$ as

$$R_{\mathrm{off}}^\Omega = \int d^4x \mathrm{tr}[\Omega\hat{D}_\mu\Omega^\dagger, \vec{H}][\Omega\hat{D}^\mu\Omega^\dagger, \vec{H}]^\dagger = \int d^4x \mathrm{tr}[\hat{D}_\mu, \Omega^\dagger\vec{H}\Omega][\hat{D}^\mu, \Omega^\dagger\vec{H}\Omega]^\dagger, \quad (2\cdot2)$$

the MA gauge fixing condition is obtained as

$$[\vec{H}, [\hat{D}_\mu, [\hat{D}^\mu, \vec{H}]]] = 0 \quad (2\cdot3)$$

from the infinitesimal gauge transformation of Ω. In the MA gauge,

$$\Phi_{\mathrm{MA}}[A_\mu(x)] \equiv [\hat{D}_\mu, [\hat{D}^\mu, \vec{H}]] \quad (2\cdot4)$$

is diagonalized, and $G \equiv \mathrm{SU}(N_c)_{\mathrm{local}}$ is reduced into $\mathrm{U}(1)^{N_c-1}_{\mathrm{local}} \times \mathrm{Weyl}_{\mathrm{global}}$, where the *global Weyl symmetry* is the subgroup of SU(N_c) relating the permutation of the basis in the fundamental representation.[7,8]

In the lattice formalism with the Euclidean metric, the MA gauge is defined by maximizing the diagonal element of the link variable $U_\mu(s) \equiv \exp\{iaeA_\mu(s)\}$,

$$R_{\rm diag}[U_\mu(\cdot)] \equiv \sum_{s,\mu} {\rm tr}\{U_\mu(s)\vec H U_\mu(s)^\dagger \vec H\}. \tag{2·5}$$

The SU(N_c) link variable is factorized corresponding to the Cartan decomposition $H \times G/H$ as

$$U_\mu(s) = M_\mu(s)u_\mu(s); \quad M_\mu(s) \equiv \exp\{i\Sigma_\alpha \theta^\alpha_\mu(s)E^\alpha\},\ u_\mu(s) \equiv \exp\{i\vec\theta_\mu(s)\cdot\vec H\}. \tag{2·6}$$

Here, the *abelian link variable* $u_\mu(s) \in H = {\rm U}(1)^{N_c-1}$ behaves as the abelian gauge field, and the off-diagonal factor $M_\mu(s) \in G/H$ behaves as the charged matter field in terms of the residual abelian gauge symmetry ${\rm U}(1)^{N_c-1}_{\rm local}$. In the lattice formalism, the abelian projection is defined by the replacement as

$$U_\mu(s) \in G \quad \rightarrow \quad u_\mu(s) \in H \tag{2·7}$$

In the MA gauge, *microscopic abelian dominance* on the link variable is observed as $U_\mu(s) \simeq u_\mu(s)$. Quantitatively, in the SU(2) lattice QCD, the *abelian projection rate*

$$R_{\rm Abel} \equiv \frac{1}{2}\langle {\rm tr}\{U_\mu(s)u_\mu(s)^\dagger\}\rangle_{\rm MA} = \frac{1}{2}\langle {\rm tr}M_\mu(s)\rangle_{\rm MA} \in [0,1] \tag{2·8}$$

is *close to unity in the MA gauge* for the whole region of β: $R_{\rm Abel}$ is above 0.88 even in the strong-coupling limit ($\beta = 0$), where the link variable is completely random before the MA gauge fixing. This is expected to be the basis of *macroscopic abelian dominance* for the infrared quantities as the string tension in the MA gauge.[7]

§3. Abelian Dominance, Monopole Dominance and Global Network of Monopole Current in MA Gauge in Lattice QCD

Abelian dominance and monopole dominance for NP-QCD (confinement, DχSB, instantons) are the remarkable facts oberved in the lattice QCD the MA gauge.[7–17] Abelian dominance means that NP-QCD is described only by the “abelian gluon”, *i.e.*, the diagonal gluon component. Monopole dominance means that the essence of NP-QCD is described only by the “monopole part” of the abelian gluon. Here, we summarize the QCD system in the MA gauge in terms of abelian dominance, monopole dominance and extraction of the relevant mode for NP-QCD.

(a) Without gauge fixing, it is difficult to extract relevant degrees of freedom for NP-QCD. All the gluon components equally contribute to NP-QCD.

(b) In the MA gauge, QCD is reduced into an abelian gauge theory including the electric current j_μ and the magnetic current k_μ. The diagonal gluon component (the abelian gluon) behaves as the abelian gauge field, and the off-diagonal gluon component (the charged gluon) behaves as the charged matter field in terms of the residual abelian gauge symmetry. In the MA gauge, the lattice QCD shows *abelian*

dominance for NP-QCD as the string tension and DχSB: only the abelian gluon is relevant for NP-QCD, while off-diagonal gluons do not contribute to NP-QCD. In the confinement phase of the lattice QCD, there appears the *global network of the monopole world-line covering the whole system in the MA gauge* .

(c) The abelian gluon can be decomposed into the "(regular) photon part" and the "(singular) monopole part", which corresponds to the separation of j_μ and k_μ.[7−15,18] The monopole part holds the monopole current k_μ, and does not include the electric current, $j_\mu \simeq 0$. On the other hand, the photon part holds the electric current j_μ only, and does not include the magnetic current, $k_\mu \simeq 0$. In the MA gauge, the lattice QCD shows *monopole dominance* for NP-QCD as the string tension, DχSB and instantons: the monopole part leads to NP-QCD, while the photon part seems trivial like QED and do not contribute to NP-QCD.

Thus, monopoles in the MA gauge can be regarded as the relevant collective mode for NP-QCD, and *formation of the global network of the monopole world-line can be regarded as "monopole condensation" in the infrared scale.* (Of course, local detailed shape of monopole world-lines is meaningless for the long-range physics of QCD !) Hence, the NP-QCD vacuum would be identified as the dual superconductor in the infrared scale in the MA gauge.

§4. Analytical Study on Abelian Dominance for Confinement

In this section, we study the connection between *microscopic abelian dominance* on the link variable and *macroscopic abelian dominance* on the infrared variable as the Wilson loop in the MA gauge, and consider the microscopic reason of abelian dominance for the confinement force. In the lattice formalism, the SU(2) link variable is factorized as $U_\mu(s) = M_\mu(s)u_\mu(s)$, according to the Cartan decomposition $G \simeq G/H \times H$. Here, $u_\mu(s) \equiv \exp\{i\tau^3\theta^3_\mu(s)\} \in H$ denotes the abelian link variable, and the off-diagonal factor $M_\mu(s) \in G/H$ is parametrized as

$$M_\mu(s) \equiv e^{i\{\tau^1\theta^1_\mu(s)+\tau^2\theta^2_\mu(s)\}} = \begin{pmatrix} \cos\theta_\mu(s) & -\sin\theta_\mu(s)e^{-i\chi_\mu(s)} \\ \sin\theta_\mu(s)e^{i\chi_\mu(s)} & \cos\theta_\mu(s) \end{pmatrix}. \qquad (4\cdot1)$$

In the MA gauge, the diagonal element $\cos\theta_\mu(s)$ in $M_\mu(s)$ is maximized by the gauge transformation as large as possible; for instance, the abelian projection rate is almost unity as $R_{\rm Abel} = \langle\cos\theta_\mu(s)\rangle_{\rm MA} \simeq 0.93$ at $\beta = 2.4$. Then, the *off-diagonal element* $e^{i\chi_\mu(s)}\sin\theta_\mu(s)$ *is forced to take a small value in the MA gauge,* and therefore the approximate treatment on the off-diagonal element would be allowed in the MA gauge. Moreover, the *angle variable* $\chi_\mu(s)$ *is not constrained* by the MA gauge-fixing condition at all, and *tends to take a random value* besides the residual $U(1)_3$ gauge degrees of freedom. Hence, $\chi_\mu(s)$ can be regarded as a *random angle variable* on the treatment of $M_\mu(s)$ in the MA gauge in a good approximation.

Let us consider the Wilson loop $\langle W_C[U_\mu(\cdot)]\rangle \equiv \langle {\rm tr}\Pi_C U_\mu(s)\rangle = \langle {\rm tr}\Pi_C\{M_\mu(s)u_\mu(s)\}\rangle$ in the MA gauge. In calculating $\langle W_C[U_\mu(\cdot)]\rangle$, the expectation value of $e^{i\chi_\mu(s)}$ in $M_\mu(s)$ vanishes as

$$\langle e^{i\chi_\mu(s)}\rangle \simeq \int_0^{2\pi} d\chi_\mu(s)\exp\{i\chi_\mu(s)\} = 0, \qquad (4\cdot2)$$

when $\chi_\mu(s)$ is assumed to be a *random angle variable.* Then, the *off-diagonal factor* $M_\mu(s)$ appearing in $\langle W_C[U_\mu(\cdot)]\rangle$ is simply reduced as a *c*-number factor, $M_\mu(s) \to \cos\theta_\mu(s)\ \mathbf{1}$, and therefore the SU(2) link variable $U_\mu(s)$ in the Wilson loop $\langle W_C[U_\mu(\cdot)]\rangle$ is simplified as a *diagonal matrix,*

$$U_\mu(s) \equiv M_\mu(s) u_\mu(s) \to \cos\theta_\mu(s) u_\mu(s). \tag{4·3}$$

Then, for the $I \times J$ rectangular C, the Wilson loop $W_C[U_\mu(\cdot)]$ in the MA gauge is approximated as

$$\begin{aligned}\langle W_C[U_\mu(\cdot)]\rangle &\equiv \langle \mathrm{tr}\mathit{\Pi}_{i=1}^L U_{\mu_i}(s_i)\rangle \simeq \langle \mathit{\Pi}_{i=1}^L \cos\theta_{\mu_i}(s_i)\cdot \mathrm{tr}\mathit{\Pi}_{j=1}^L u_{\mu_j}(s_j)\rangle_{\mathrm{MA}} \\ &= \langle \exp\{\sum_{i=1}^L \ln(\cos\theta_{\mu_i}(s_i))\}\rangle_{\mathrm{MA}}\ \langle W_C[u_\mu(\cdot)]\rangle_{\mathrm{MA}} \\ &\simeq \exp\{L\langle \ln(\cos\theta_\mu(\cdot))\rangle_{\mathrm{MA}}\}\ \langle W_C[u_\mu(\cdot)]\rangle_{\mathrm{MA}},\end{aligned} \tag{4·4}$$

where $L \equiv 2(I+J)$ denotes the perimeter length and $W_C[u_\mu(\cdot)] \equiv \mathrm{tr}\mathit{\Pi}_{i=1}^L u_{\mu_i}(s_i)$ the abelian Wilson loop. Here, we have replaced $\sum_{i=1}^L \ln\{\cos(\theta_{\mu_i}(s_i)\}$ by its average $L\langle \ln\{\cos\theta_\mu(\cdot)\}\rangle_{\mathrm{MA}}$ *in a statistical sense.*

In this way, we derive a simple estimation as

$$W_C^{\mathrm{off}} \equiv \langle W_C[U_\mu(\cdot)]\rangle / \langle W_C[u_\mu(\cdot)]\rangle_{\mathrm{MA}} \simeq \exp\{L\langle \ln(\cos\theta_\mu(\cdot))\rangle_{\mathrm{MA}}\} \tag{4·5}$$

for the *contribution of the off-diagonal gluon element to the Wilson loop.* From this analysis, the contribution of off-diagonal gluons to the Wilson loop is expected to obey the *perimeter law* in the MA gauge for large loops, where the statistical treatment would be accurate.

In the lattice QCD, we find that W_C^{off} seems to obey the *perimeter law* for the Wilson loop with $I, J \ge 2$ in the MA gauge. We find also that the behavior on W_C^{off} as the function of L is well reproduced by the above estimation with the *microscopic information* on the diagonal factor $\cos\theta_\mu(s)$ as $\langle \ln\{\cos_\mu(s)\}\rangle_{\mathrm{MA}} \simeq -0.082$ for $\beta = 2.4$.

Thus, the off-diagonal contribution W_C^{off} to the Wilson loop obeys the perimeter law in the MA gauge, and therefore the *abelian Wilson loop* $\langle W_C[u_\mu(\cdot)]\rangle_{\mathrm{MA}}$ should obey the *area law* as well as the SU(2) Wilson loop $W_C[U_\mu(\cdot)]$. From Eq.(4.5), the off-diagonal contribution to the string tension vanishes as

$$\begin{aligned}\sigma_{\mathrm{SU(2)}} - \sigma_{\mathrm{Abel}} &\equiv -\lim_{R,T\to\infty} \frac{1}{RT} \ln\langle W_{R\times T}[U_\mu(\cdot)]\rangle + \lim_{R,T\to\infty} \frac{1}{RT} \ln\langle W_{R\times T}[u_\mu(\cdot)]\rangle_{\mathrm{MA}} \\ &\simeq -2\langle \ln\{\cos\theta_\mu(s)\}\rangle_{\mathrm{MA}} \lim_{R,T\to\infty} \frac{R+T}{RT} = 0.\end{aligned} \tag{4·6}$$

Thus, *abelian dominance for the string tension*, $\sigma_{\mathrm{SU(2)}} = \sigma_{\mathrm{Abel}}$, can be proved in the MA gauge by replacing the off-diagonal angle variable $\chi_\mu(s)$ as a random variable. This estimation indicates also that the *finite size effect* on R and T leads to the deviation between $\sigma_{\mathrm{SU(2)}}$ and σ_{Abel}, *i.e.*, $\sigma_{\mathrm{SU(2)}} > \sigma_{\mathrm{Abel}}$.

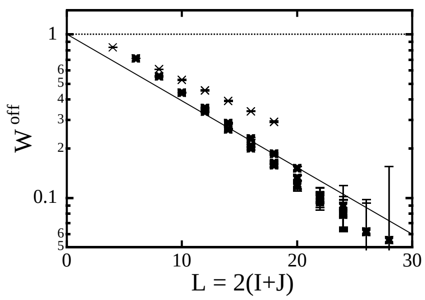


Fig. 1. The comparison between the lattice data and the analytical estimation of $W_C^{\rm off} \equiv \langle W_C[U_\mu(\cdot)]\rangle / \langle W_C[u_\mu(\cdot)]\rangle$ as the function of the perimeter $L \equiv 2(I+J)$ in the MA gauge. The cross ($\times$) denotes the lattice date at $\beta = 2.4$, and the straight line denotes the theoretical estimation of $W_C^{\rm off} = \exp\{L\langle \ln(\cos\theta_\mu(s))\rangle_{\rm MA}\}$ with the microscopic input $\langle \ln\{\cos\theta_\mu(s)\}\rangle_{\rm MA} \simeq -0.082$ at $\beta = 2.4$. The off-diagonal gluon contribution $W_C^{\rm off}$ seems to obey the *perimeter law* for $I, J \geq 2$ (thick cross symbols).

§5. Origin of Abelian Dominance : Effective Charged-Gluon Mass induced in MA Gauge

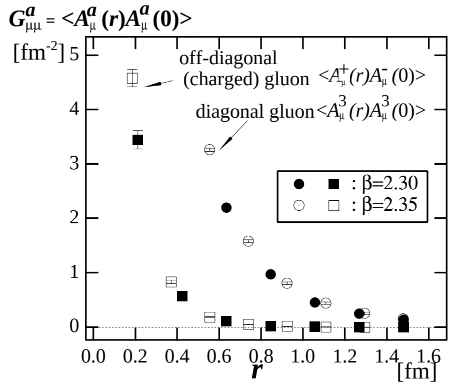
In the MA gauge, only the diagonal gluon component is relevant for the infrared quantities like the string tension and the chiral condensate, and it is regarded as abelian dominance for NP-QCD. In this section, we study the *origin of abelian dominance in the MA gauge.*

As a possible physical interpretation, abelian dominance can be expressed as *generation of the effective mass m_{ch} of the off-diagonal (charged) gluon by the MA gauge fixing* in the QCD partition functional,[2]

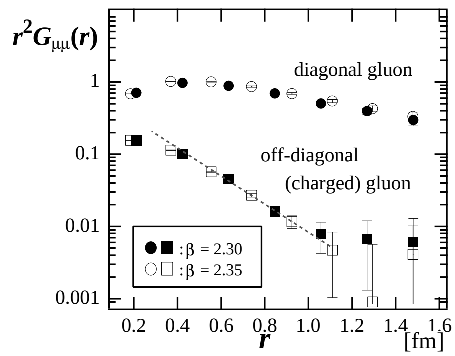
$$
\begin{aligned}
Z_{\rm QCD}^{\rm MA} &= \int DA_\mu \exp\{iS_{\rm QCD}[A_\mu]\}\delta(\Phi_{\rm MA}^{\pm}[A_\mu])\Delta_{\rm PF}[A_\mu] \\
&\simeq \int DA_\mu^3 \exp\{iS_{\rm eff}[A_\mu^3]\} \int DA_\mu^{\pm} \exp\{i\int d^4x\ m_{ch}^2 A_\mu^+ A_-^\mu\}\mathcal{F}[A_\mu], \quad (5\cdot1)
\end{aligned}
$$

where $\Delta_{\rm FP}$ is the Faddeev-Popov determinant, $S_{\rm eff}[A_\mu^3]$ the abelian effective action and $\mathcal{F}[A_\mu]$ a smooth functional. In fact, if the MA gauge fixing induces the effective mass m_{ch} of off-diagonal (charged) gluons, the charged gluon propagation is limited within the short-range region as $r \lesssim m_{ch}^{-1}$, and hence off-diagonal gluons cannot contribute to the long-distance physics in the MA gauge, which provides the origin of abelian dominance for NP-QCD.

Here, using the SU(2) lattice QCD in the Euclidean metric, we study the gluon propagator $G_{\mu\nu}^{ab}(x-y) \equiv \langle A_\mu^a(x) A_\nu^b(y)\rangle$ in the MA gauge with respect to the interaction range and strength.[2,19] As for the residual U(1) gauge symmetry, we impose the U(1) Landau gauge fixing to extract most continuous gauge configuration and to compare with the continuum theory. In particular, the scalar combination $G_{\mu\mu}^a(r) \equiv \sum_{\mu=1}^4 \langle A_\mu{}^a(r) A_\mu{}^a(0)\rangle$ $(a = 1,2,3)$ is useful to observe the interaction range of the gluon, because it depends only on the four-dimensional Euclidean radial coordinate $r \equiv (x_\mu x_\mu)^{\frac{1}{2}}$.



(a)



(b)

Fig. 2. (a) The scalar correlation $G^a_{\mu\mu}(r)$ of the gluon propagator as the function of the 4-dimensional distance r in the MA gauge in the SU(2) lattice QCD with $12^3 \times 24$ and $\beta = 2.3, 2.35$. In the MA gauge, the off-diagonal (charged) gluon propagates within the short-range region $r \lesssim 0.4$ fm, and cannot contribute to the long-range physics. (b) The logarithmic plot for the scalar correlation $r^2 G^a_{\mu\mu}(r)$. The charged-gluon propagator behaves as the Yukawa-type function, $G_{\mu\mu} \sim \frac{\exp(-m_{ch} r)}{r^2}$. The effective mass of the charged gluon can be estimated as $m_{ch} \simeq 0.9$GeV from the slope of the dotted line.

We calculate the gluon propagator $G^a_{\mu\mu}(r)$ in the MA gauge using the SU(2) lattice QCD with $12^3 \times 24$ and $\beta = 2.3, 2.35$. In the MA gauge, the off-diagonal (charged) gluon propagates within the short-range region $r \lesssim 0.4$ fm, so that it cannot contribute to the long-range physics, although the charged-gluon effect appears at the short distance as $r \lesssim 0.4$ fm. On the other hand, the diagonal gluon propagates over the long distance and influences the long-range physics. Thus, we find *abelian dominance for the gluon propagator* : only the diagonal gluon is relevant at the infrared scale in the MA gauge. This is the *origin of abelian dominance for the long-distance physics or NP-QCD.*[2,19]

Since the propagator of the massive gauge boson with mass M behaves as the Yukawa-type function $G_{\mu\mu}(r) = \frac{3}{4\pi^2} \frac{1}{r^2} \exp(-Mr)$, the effective mass m_{ch} of the charged gluon can be evaluated from the slope of the logarithmic plot of $r^2 G^{+-}_{\mu\mu}(r) \sim \exp(-m_{ch} r)$ as shown in Fig.3. The charged gluon behaves as a massive particle at the long distance, $r \gtrsim 0.4$ fm. We obtain the *effective mass of the charged gluon* as $m_{ch} \simeq 0.9$ GeV, which provides the *critical scale on abelian dominance.*

§6. Dual Wilson Loop, Inter-Monopole Potential and Evidence of Dual Higgs Mechanism (Monopole Condensation)

In this section, we study the dual Higgs mechanism by monopole condensation in the NP-QCD vacuum in the field-theoretical manner. Since QCD is described by the "electric variable" as quarks and gluons, the "electric sector" of QCD has been well studied with the Wilson loop or the inter-quark potential, however, the "magnetic sector" of QCD is hidden and still unclear. To investigate the magnetic sector

directly, it is useful to introduce the "dual (magnetic) variable" as the *dual gluon* B_μ, similarly in the dual Ginzburg-Landau (DGL) theory.[4,5,20−24] The dual gluon B_μ is the dual partner of the abelian gluon and directly couples with the magnetic current k_μ. In particular, in the absence of the electric current, $\partial_\mu F^{\mu\nu} = j^\nu = 0$, the dual gluon B_μ can be introduced as the regular field satisfying $\partial_\mu B_\nu - \partial_\nu B_\mu = {}^*F_{\mu\nu}$ and the dual Bianchi identity, $\partial^{\mu *}(\partial \wedge B)_{\mu\nu} = 0$, and therefore the argument on monopole condensation becomes transparent.[20,23]

As was mentioned in Section 3, the monopole part in the MA gauge and does not include the electric current, $j_\mu \simeq 0$, and holds the essence of NP-QCD, which is of interest. Then, it is wise to consider the monopole part for the transparent argument on the dual Higgs mechanism or monopole condensation. In terms of the dual Higgs mechanism, the inter-monopole potential is expected to be short-range Yukawa-type, and the dual gluon B_μ becomes massive in the monopole-condensed vacuum.[4,5,20] We define *the dual Wilson loop* W_D as the line-integral of the dual gluon B_μ along a loop C,

$$W_D(C) \equiv \exp\{i\frac{e}{2}\oint_C dx_\mu B^\mu\} = \exp\{i\frac{e}{2}\iint d\sigma_{\mu\nu}{}^*F^{\mu\nu}\}, \qquad (6\cdot 1)$$

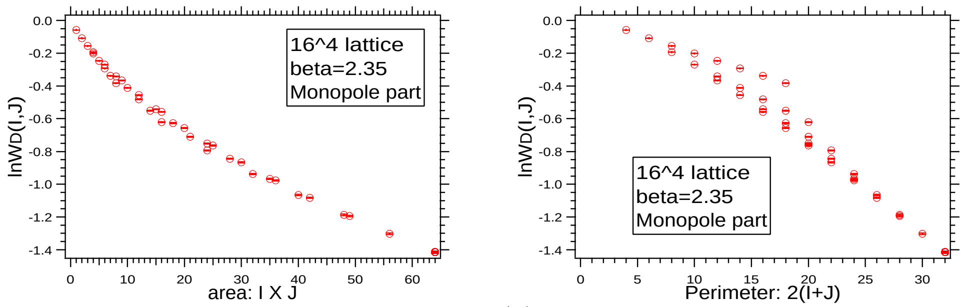
which is the *dual version of the abelian Wilson loop* $W_{\rm Abel}(C) \equiv \exp\{i\frac{e}{2}\oint_C dx_\mu A^\mu\} = \exp\{i\frac{e}{2}\iint d\sigma_{\mu\nu}F^{\mu\nu}\}$ in the SU(2) case.[25] Here, we have set the test monopole charge as $e/2$ considering this duality correspondence. The potential between the monopole and the anti-monopole is derived from the dual Wilson loop as

$$V_M(R) = -\lim_{T\to\infty}\frac{1}{T}\ln\langle W_D(R,T)\rangle. \qquad (6\cdot 2)$$

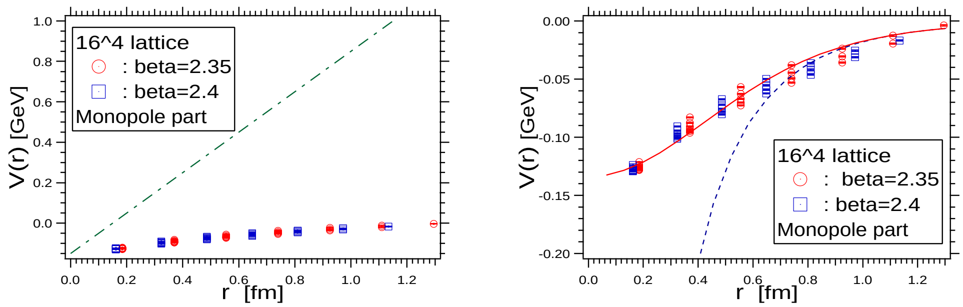
Using the SU(2) lattice QCD in the MA gauge, we study the dual Wilson loop and the inter-monopole potential in the monopole part.[25] The dual Wilson loop $\langle W_D(I,J)\rangle$ seems to obey the *perimeter law* rather than the area law for $I, J \geq 2$. The inter-monopole potential is short ranged and flat in comparison with the inter-quark potential as shown in Fig.4.

At the long distance, the inter-monopole potential can be fitted by the simple Yukawa potential $V_M(r) = -\frac{(e/2)^2}{4\pi}\frac{e^{-m_B r}}{r}$. The dual gluon mass is estimated as $m_B \simeq$ 0.5GeV, which is consistent with the DGL theory.[4,5,20−23] *The mass generation of the dual gluon B_μ would provide the direct evidence of the dual Higgs mechanism by monopole condensation at the infrared scale in the NP-QCD vacuum.*

In the whole region of r including the short distance, the inter-monopole potential seems to be fitted by the Yukawa-type potential with the effective size R of the QCD-monopole as shown in Fig.4.
The fitting on the global shape of $V_M(r)$ suggests the dual gluon mass as $m_B \simeq$ 0.5GeV and the *effective monopole size* $R \simeq 0.35$fm, which would provide the *critical scale for NP-QCD in terms of the dual Higgs theory as the local field theory.*



(a)



(b)

Fig. 3. (a) The dual Wilson loop $\langle W_D(I,J)\rangle$ v.s. its area $(I\times J)$ and its perimeter $2(I+J)$ in the monopole part in the MA gauge in the SU(2) lattice QCD with 16^4 and $\beta=2.35$. $\langle W_D(I,J)\rangle$ seems to obey the *perimeter law* rather than the area law for $I,J\geq 2$ (thick cross symbols). (b) The inter-monopole potential $V_M(r)$ in the monopole part in the MA gauge extracted from $\langle W_D(I,J)\rangle$. r is the 3-dimensional distance between the monopole and the anti-monopole. The dashed-dotted line denotes the linear part of the inter-quark potential in the left figure. In the right figure, the dashed curve and the solid curve denote the simple Yukawa potential and the Yukawa-type potential with the effective monopole size, respectively.

§7. Origin of Strong Correlation between Monopoles and Instantons : Large Gluon-Field Fluctuation around Monopoles

There is no point-like monopole in QED, because the QED action diverges around the monopole. The QCD-monopole also accompanies the large fluctuation of the abelian action density inevitably. In this section, we study the action density around the QCD-monopole in the MA gauge using the SU(2) lattice QCD.[26]

From the SU(2) plaquette $P_{\mu\nu}^{\rm SU(2)}(s)$ and the abelian plaquette $P_{\mu\nu}^{\rm Abel}(s)$, we define the "SU(2) action density" $S_{\mu\nu}^{\rm SU(2)}(s)\equiv 1-\frac{1}{2}{\rm tr}P_{\mu\nu}^{\rm SU(2)}(s)$, the "abelian action density" $S_{\mu\nu}^{\rm Abel}(s)\equiv 1-\frac{1}{2}{\rm tr}P_{\mu\nu}^{\rm Abel}(s)$ and the "off-diagonal action density"

$$S_{\mu\nu}^{\rm off}(s)\equiv S_{\mu\nu}^{\rm SU(2)}(s)-S_{\mu\nu}^{\rm Abel}(s), \qquad (7\cdot1)$$

which is *not positive definite*. In the lattice formalism, the monopole current $k_\mu(s)$ is defined on the dual link, and there are 12 links around the monopole. To investigate the "local quantity around the monopole", we define the "local average over the

neighboring 12 links around the dual link",

$$\bar{S}(s,\mu) \equiv \frac{1}{12} \sum_{\alpha\beta\gamma} \sum_{m=0}^{1} |\varepsilon_{\mu\alpha\beta\gamma}| S_{\alpha\beta}(s+m\hat{\gamma}). \tag{7·2}$$

From the total ensemble $\{\bar{S}(s,\mu)\}_{s,\mu}$, we extract the sub-ensemble $\{\bar{S}(s,\mu)\}_{s,\mu}$ *around the monopole* in the lattice QCD. We show in Fig.5(b) the probability distribution of $\bar{S}_{\rm SU(2)}$, $\bar{S}_{\rm Abel}$ and $\bar{S}_{\rm off}$ around the monopole in the MA gauge.

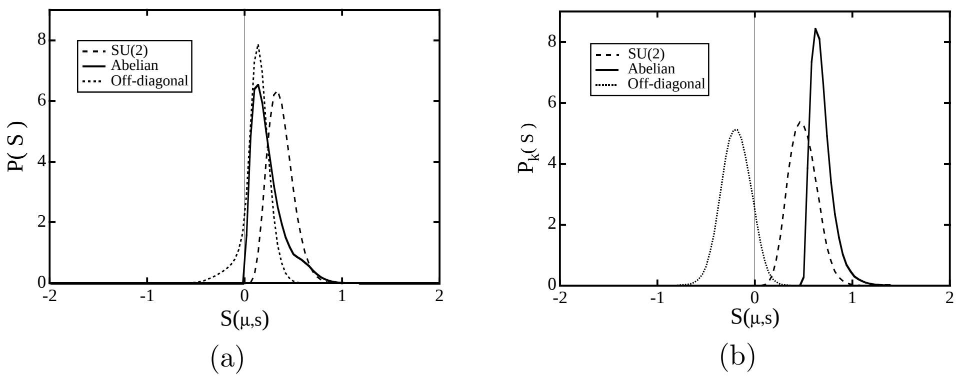


Fig. 4. (a) The total probability distribution $P(\bar{S})$ and (b) The probability distribution $P_k(\bar{S})$ around the monopole for SU(2) action density $\bar{S}_{\rm SU(2)}$ (dashed curve), abelian action density $\bar{S}_{\rm Abel}$ (solid curve) and off-diagonal part $\bar{S}_{\rm off}$ (dotted curve) in the MA gauge at $\beta = 2.4$ on 16^4 lattice. Large cancellation between $\bar{S}_{\rm Abel}$ and $\bar{S}_{\rm off}$ is observed around the QCD-monopole.

Around the monopole in the MA gauge, a *large fluctuation* is observed both in $S_{\rm Abel}$ and in $S_{\rm off}$, however, the total QCD action $\bar{S}_{\rm SU(2)}$ is kept to be small relatively, owing to *large cancellation* between $\bar{S}_{\rm Abel}$ and $\bar{S}_{\rm off}$ as shown in Fig.5(b).[26] Thus, off-diagonal gluons play the essential role to appearance of QCD-monopoles to keep the total QCD action finite. However, in the infrared scale, off-diagonal gluons become irrelevant, while large abelian-gluon fluctuations originated from QCD-monopoles remain to be relevant in the MA gauge.

Even in the MA gauge, off-diagonal gluons largely remain around the QCD-monopole, which would provide the *effective monopole size* R as the critical scale of abelian projected QCD (AP-QCD), because off-diagonal gluons become visible and AP-QCD should be modified at the shorter scale than R, which resembles the structure of the 't Hooft-Polyakov monopole. The concentration of off-diagonal gluons around monopoles leads to the *local correlation between monopoles and instantons*: instantons appear around the monopole world-line in the MA gauge, because instantons need full SU(2) gluon components for existence.[7,8,13−15,23,24,27−29]

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